opposed by the motor's self-generated emf, called the **back emf** of the motor. (See Figure 23.25.)

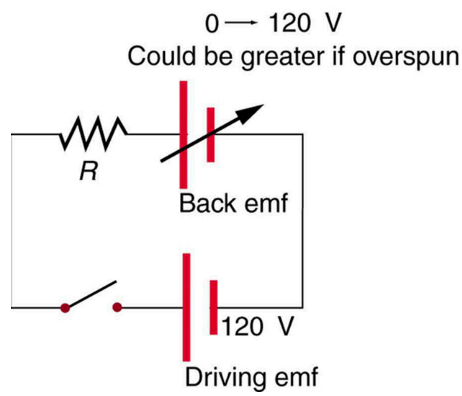

Figure 23.25 The coil of a DC motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the one driving the motor. Back emf is zero when the motor is not turning, and it increases proportionally to the motor's angular velocity.

Back emf is the generator output of a motor, and so it is proportional to the motor's angular velocity ω. It is zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster and faster, the back emf grows, always opposing the driving emf, and reduces the voltage across the coil and the amount of current it draws. This effect is noticeable in a number of situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the IR drop produced in feeder lines by the large current drawn by the motor. When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil, $P = I^2R$), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it will increase its angular velocity ω until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Consider, for example, the motor coils represented in Figure 23.25. The coils have a $0.400\ \Omega$ equivalent resistance and are driven by a 48.0 V emf. Shortly after being turned on, they draw a current $I = V/R = (48.0\ \text{V})/(0.400\ \Omega) = 120\ \text{A}$ and, thus, dissipate $P = I^2R = 5.76\ \text{kW}$ of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is 40.0 V. Then at operating speed, the total voltage across the coils is 8.0 V (48.0 V minus the 40.0 V back emf), and the current drawn is $I = V/R = (8.0\ \text{V})/(0.400\ \Omega) = 20\ \text{A}$. Under normal load, then, the power dissipated is $P = IV = (20\ \text{A})/(8.0\ \text{V}) = 160\ \text{W}$. The latter will not cause a problem for this motor, whereas the former 5.76 kW would burn out the coils if sustained.

23.7 Transformers

Transformers do what their name implies—they transform voltages from one value to another (The term voltage is used rather than emf, because transformers have internal resistance). For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in Figure 23.26) that changes 120 V or 240 V AC into whatever voltage the device uses. Transformers are also used at several points in the power distribution systems, such as illustrated in Figure 23.27. Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.

Figure 23.26 The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V AC. Most are in the 3 to 12 V range. (credit: Shop Xtreme)

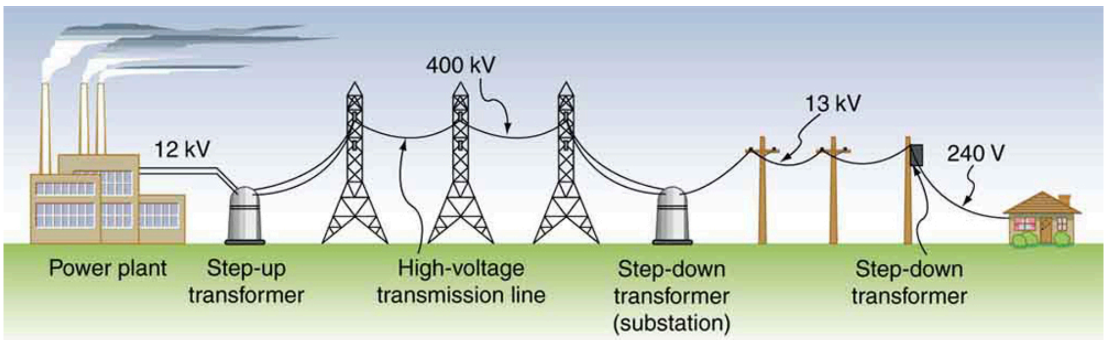

Figure 23.27 Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered in this text—see Figure 23.28—is based on Faraday's law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the *primary* and *secondary coils*. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.

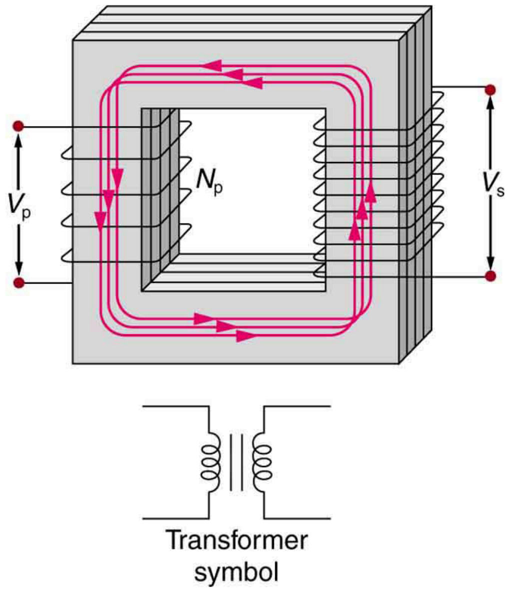

Figure 23.28 A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.

For the simple transformer shown in Figure 23.28, the output voltage $V_{\rm s}$ depends almost entirely on the input voltage $V_{\rm p}$ and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage $V_{\rm s}$ to be

$$V_{\rm s} = -N_{\rm s}\frac{\Delta\Phi}{\Delta t}, \tag{23.24}$$

where $N_{\rm s}$ is the number of loops in the secondary coil and $\Delta\Phi/\Delta t$ is the rate of change of magnetic flux. Note that the output voltage equals the induced emf ($V_{\rm s} = \text{emf}_{\rm s}$), provided coil resistance is small (a reasonable assumption for transformers). The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on either side. The input primary voltage $V_{\rm p}$ is also related to changing flux by

$$V_p = -N_{\rm p}\frac{\Delta\Phi}{\Delta t}. \tag{23.25}$$

The reason for this is a little more subtle. Lenz's law tells us that the primary coil opposes the change in flux caused by the input voltage $V_{\rm p}$, hence the minus sign (This is an example of *self-inductance*, a topic to be explored in some detail in later sections). Assuming negligible coil resistance, Kirchhoff's loop rule tells us that the induced emf exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

$$\frac{V_{\rm s}}{V_{\rm p}} = \frac{N_{\rm s}}{N_{\rm p}}. \tag{23.26}$$

This is known as the **transformer equation**, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A **step-up transformer** is one that increases voltage, whereas a **step-down transformer** decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This is nearly true in practice—transformer efficiency often exceeds 99%. Equating the power input and output,

$$P_{\rm p} = I_{\rm p}V_{\rm p} = I_{\rm s}V_{\rm s} = P_{\rm s}. \tag{23.27}$$

Rearranging terms gives

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}. \tag{23.28}$$

Combining this with $\frac{V_s}{V_p} = \frac{N_s}{N_p}$, we find that

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \tag{23.29}$$

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

EXAMPLE 23.5

Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s, the number of loops in the secondary, and enter the known values. This gives

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (50)\frac{100{,}000\text{ V}}{120\text{ V}} = 4.17 \times 10^4. \end{aligned} \tag{23.30}$$

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

We can similarly find the output current of the secondary by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_s and entering known values. This gives

$$\begin{aligned} I_s &= I_p \frac{N_p}{N_s} \\ &= (10.00\text{ A})\frac{50}{4.17\times10^4} = 12.0\text{ mA}. \end{aligned} \tag{23.31}$$

Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is $P_p = I_p V_p = (10.00\text{ A})(120\text{ V}) = 1.20\text{ kW}$. This equals the power output $P_p = I_s V_s = (12.0\text{ mA})(100\text{ kV}) = 1.20\text{ kW}$, as we assumed in the derivation of the equations used.

The fact that transformers are based on Faraday's law of induction makes it clear why we cannot use transformers to change DC voltages. If there is no change in primary voltage, there is no voltage induced in the secondary. One possibility is to connect DC to the primary coil through a switch. As the switch is opened and closed, the secondary produces a voltage like that in Figure 23.29. This is not really a practical alternative, and AC is in common use wherever it is necessary to increase or decrease voltages.

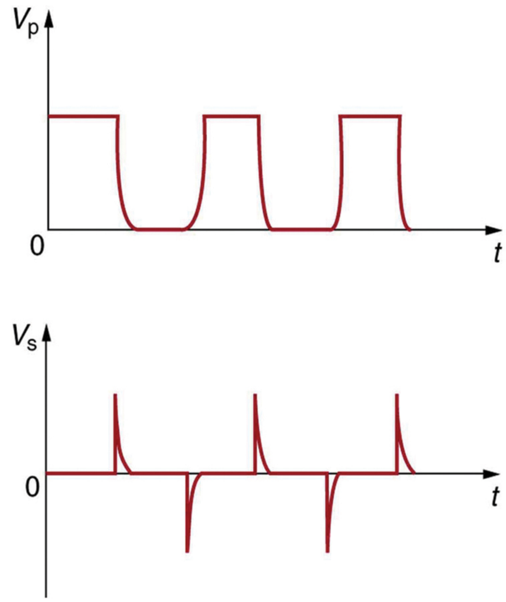

Figure 23.29 Transformers do not work for pure DC voltage input, but if it is switched on and off as on the top graph, the output will look something like that on the bottom graph. This is not the sinusoidal AC most AC appliances need.

EXAMPLE 23.6

Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries (total emf of 12.5 V DC) needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s and entering known values gives

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (200)\frac{15.0\text{ V}}{120\text{ V}} = 25. \end{aligned} \tag{23.32}$$

Strategy and Solution for (b)

The current input can be obtained by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_p and entering known values. This gives

$$\begin{aligned} I_p &= I_s \frac{N_s}{N_p} \\ &= (16.0\text{ A})\frac{25}{200} = 2.00\text{ A}. \end{aligned} \tag{23.33}$$

Discussion

The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in ($P_p = P_s$)—reasonable for good transformers. In this case the primary and secondary power is 240 W. (Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

Transformers have many applications in electrical safety systems, which are discussed in Electrical Safety: Systems and Devices.

PHET EXPLORATIONS

Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

Click to view content (https://phet.colorado.edu/sims/faraday/generator-600.png)

Figure 23.30

Generator (https://phet.colorado.edu/en/simulation/legacy/generator)

23.8 Electrical Safety: Systems and Devices

Electricity has two hazards. A **thermal hazard** occurs when there is electrical overheating. A **shock hazard** occurs when electric current passes through a person. Both hazards have already been discussed. Here we will concentrate on systems and devices that prevent electrical hazards.

Figure 23.31 shows the schematic for a simple AC circuit with no safety features. This is not how power is distributed in practice. Modern household and industrial wiring requires the **three-wire system**, shown schematically in Figure 23.32, which has several safety features. First is the familiar *circuit breaker* (or *fuse*) to prevent thermal overload. Second, there is a protective *case* around the appliance, such as a toaster or refrigerator. The case's safety feature is that it prevents a person from touching exposed wires and coming into electrical contact with the circuit, helping prevent shocks.

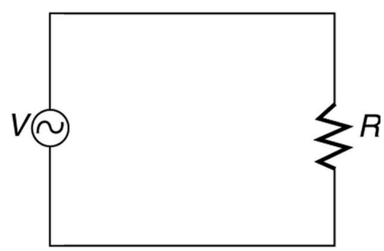

Figure 23.31 Schematic of a simple AC circuit with a voltage source and a single appliance represented by the resistance R. There are no safety features in this circuit.

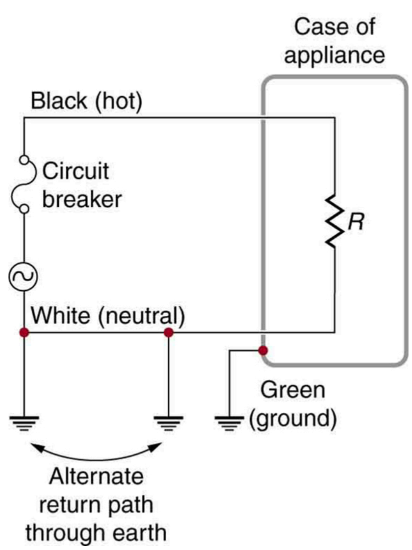

Figure 23.32 The three-wire system connects the neutral wire to the earth at the voltage source and user location, forcing it to be at zero volts and supplying an alternative return path for the current through the earth. Also grounded to zero volts is the case of the appliance. A